

# Robust Wavelet-based Assessment of Scaling with Applications

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## **APPENDIX B: Simulation Results from Section 3.3**

We present here the rest of the simulation results described in Section 3.3. The paper contains tables for  $H = 0.5$ , here we provide the results for  $H = 0.3, 0.4, 0.6$  and  $0.7$ .

Tables 7 through 14 summarize estimations of  $H$  and MSEs under four different wavelet filters, in the non-contaminated and contaminated 1-D fBm realizations. Tables 15 through 22 summarize the same results from 2-D fBm realizations. As explained earlier, cells highlighted with underline represent lowest bias, and the gray cells indicate the cases with lowest MSE.

Table 7: Estimations of  $H$  and MSEs for  $H = 0.3$  under four different wavelet filters, in the non-contaminated case.

		Haar	Coiflet4	Daub6	Symmlet8
OLS	$H$	<b>0.223</b>	<b>0.253</b>	<b>0.255</b>	<b>0.251</b>
	$MSE$	0.013	0.012	0.010	0.011
AV	$H$	<b>0.191</b>	<b>0.218</b>	<b>0.219</b>	<b>0.216</b>
	$MSE$	0.017	0.011	0.011	0.011
TT	$H$	<b>0.230</b>	<b>0.259</b>	<b>0.260</b>	<b>0.256</b>
	$MSE$	0.010	0.007	0.007	0.007

Table 8: Estimations of  $H$  and MSEs for  $H = 0.3$  under four different wavelet filters, in the contaminated case.

		Haar	Coiflet4	Daub6	Symmlet8
OLS	$H$	<b>0.314</b>	<b>0.325</b>	<b>0.323</b>	<b>0.332</b>
	$MSE$	0.013	0.013	0.013	0.014
AV	$H$	<b>0.232</b>	<b>0.251</b>	<b>0.246</b>	<b>0.252</b>
	$MSE$	0.010	0.007	0.009	0.007
TT	$H$	<b>0.288</b>	<b>0.302</b>	<b>0.302</b>	<b>0.309</b>
	$MSE$	0.008	0.006	0.007	0.006

Table 9: Estimations of  $H$  and MSEs for  $H = 0.4$  under four different wavelet filters, in the non-contaminated case.

		Haar	Coiflet4	Daub6	Symmlet8
OLS	$H$	<b>0.348</b>	<b>0.345</b>	<b>0.336</b>	<b>0.402</b>
	$MSE$	0.011	0.009	0.015	0.008
AV	$H$	<b>0.329</b>	<b>0.331</b>	<b>0.322</b>	<b>0.353</b>
	$MSE$	0.010	0.008	0.012	0.007
TT	$H$	<b>0.364</b>	<b>0.363</b>	<b>0.354</b>	<b>0.397</b>
	$MSE$	0.007	0.005	0.009	0.005

Table 10: Estimations of  $H$  and MSEs for  $H = 0.4$  under four different wavelet filters, in the contaminated case.

		Haar	Coiflet4	Daub6	Symmlet8
OLS	$H$	<b>0.452</b>	<b>0.449</b>	<b>0.442</b>	<b>0.446</b>
	$MSE$	0.016	0.014	0.015	0.015
AV	$H$	<b>0.371</b>	<b>0.370</b>	<b>0.374</b>	<b>0.367</b>
	$MSE$	0.007	0.006	0.007	0.006
TT	$H$	<b>0.416</b>	<b>0.423</b>	<b>0.426</b>	<b>0.415</b>
	$MSE$	0.008	0.007	0.010	0.007

Table 11: Estimations of  $H$  and MSEs for  $H = 0.6$  under four different wavelet filters, in the non-contaminated case.

		Haar	Coiflet4	Daub6	Symmlet8
OLS	$H$	<b>0.534</b>	<b>0.526</b>	<b>0.544</b>	<b>0.554</b>
	$MSE$	0.016	0.014	0.012	0.011
AV	$H$	<b>0.528</b>	<b>0.462</b>	<b>0.520</b>	<b>0.532</b>
	$MSE$	0.012	0.027	0.013	0.010
TT	$H$	<b>0.557</b>	<b>0.511</b>	<b>0.555</b>	<b>0.566</b>
	$MSE$	0.010	0.015	0.009	0.007

Table 12: Estimations of  $H$  and MSEs for  $H = 0.6$  under four different wavelet filters, in the contaminated case.

		Haar	Coiflet4	Daub6	Symmlet8
OLS	$H$	<b>0.628</b>	<b>0.596</b>	<b>0.598</b>	<b>0.612</b>
	$MSE$	0.016	0.014	0.012	0.012
AV	$H$	<b>0.568</b>	<b>0.457</b>	<b>0.530</b>	<b>0.494</b>
	$MSE$	0.007	0.028	0.011	0.017
TT	$H$	<b>0.615</b>	<b>0.531</b>	<b>0.576</b>	<b>0.561</b>
	$MSE$	0.008	0.012	0.008	0.008

Table 13: Estimations of  $H$  and MSEs for  $H = 0.7$  under four different wavelet filters, in the non-contaminated case.

		Haar	Coiflet4	Daub6	Symmlet8
OLS	$H$	<b>0.662</b>	<b>0.572</b>	<b>0.567</b>	<b>0.592</b>
	$MSE$	0.008	0.027	0.028	0.021
AV	$H$	<b>0.653</b>	<b>0.507</b>	<b>0.553</b>	<b>0.527</b>
	$MSE$	0.006	0.051	0.030	0.041
TT	$H$	<b>0.683</b>	<b>0.556</b>	<b>0.585</b>	<b>0.577</b>
	$MSE$	0.005	0.032	0.022	0.025

Table 14: Estimations of  $H$  and MSEs for  $H = 0.7$  under four different wavelet filters, in the contaminated case.

		Haar	Coiflet4	Daub6	Symmlet8
OLS	$H$	<b>0.733</b>	<b>0.603</b>	<b>0.616</b>	<b>0.636</b>
	$MSE$	0.011	0.020	0.017	0.014
AV	$H$	<b>0.678</b>	<b>0.438</b>	<b>0.536</b>	<b>0.489</b>
	$MSE$	0.005	0.075	0.032	0.053
TT	$H$	<b>0.719</b>	<b>0.522</b>	<b>0.584</b>	<b>0.562</b>
	$MSE$	0.007	0.038	0.020	0.027

Table 15: Estimations of  $H$  and MSEs for  $H = 0.3$  from three directions; under four different wavelet filters, in the non-contaminated case.

		Haar			Coiflet4		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b><u>0.213</u></b>	<b><u>0.266</u></b>	<b><u>0.260</u></b>	<b><u>0.252</u></b>	<b><u>0.319</u></b>	<b><u>0.317</u></b>
	MSE	0.008	0.002	0.003	0.003	0.002	0.002
AV	$H$	<b><u>0.111</u></b>	<b><u>0.231</u></b>	<b><u>0.227</u></b>	<b><u>0.168</u></b>	<b><u>0.249</u></b>	<b><u>0.249</u></b>
	MSE	0.036	0.005	0.006	0.018	0.003	0.003
TT	$H$	<b><u>0.141</u></b>	<b><u>0.243</u></b>	<b><u>0.238</u></b>	<b><u>0.193</u></b>	<b><u>0.263</u></b>	<b><u>0.262</u></b>
	MSE	0.025	0.004	0.004	0.012	0.002	0.002

  

		Daub6			Symmlet8		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b><u>0.244</u></b>	<b><u>0.316</u></b>	<b><u>0.317</u></b>	<b><u>0.253</u></b>	<b><u>0.316</u></b>	<b><u>0.325</u></b>
	MSE	0.004	0.002	0.002	0.003	0.002	0.003
AV	$H$	<b><u>0.160</u></b>	<b><u>0.281</u></b>	<b><u>0.282</u></b>	<b><u>0.166</u></b>	<b><u>0.254</u></b>	<b><u>0.258</u></b>
	MSE	0.020	0.001	0.001	0.018	0.002	0.002
TT	$H$	<b><u>0.186</u></b>	<b><u>0.293</u></b>	<b><u>0.293</u></b>	<b><u>0.192</u></b>	<b><u>0.267</u></b>	<b><u>0.271</u></b>
	MSE	0.013	0.001	0.001	0.012	0.002	0.001

Table 16: Estimations of  $H$  and MSEs for  $H = 0.3$  from three directions; under four different wavelet filters, in the contaminated case.

		Haar			Coiflet4		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b><u>0.312</u></b>	<b><u>0.362</u></b>	<b><u>0.356</u></b>	<b><u>0.356</u></b>	<b><u>0.409</u></b>	<b><u>0.424</u></b>
	MSE	0.002	0.006	0.005	0.005	0.014	0.018
AV	$H$	<b><u>0.122</u></b>	<b><u>0.244</u></b>	<b><u>0.242</u></b>	<b><u>0.181</u></b>	<b><u>0.260</u></b>	<b><u>0.260</u></b>
	MSE	0.032	0.003	0.004	0.014	0.002	0.002
TT	$H$	<b><u>0.164</u></b>	<b><u>0.265</u></b>	<b><u>0.264</u></b>	<b><u>0.217</u></b>	<b><u>0.283</u></b>	<b><u>0.285</u></b>
	MSE	0.019	0.002	0.002	0.007	0.001	0.001

  

		Daub6			Symmlet8		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b><u>0.346</u></b>	<b><u>0.418</u></b>	<b><u>0.414</u></b>	<b><u>0.353</u></b>	<b><u>0.415</u></b>	<b><u>0.413</u></b>
	MSE	0.003	0.017	0.015	0.004	0.016	0.015
AV	$H$	<b><u>0.175</u></b>	<b><u>0.299</u></b>	<b><u>0.294</u></b>	<b><u>0.182</u></b>	<b><u>0.265</u></b>	<b><u>0.266</u></b>
	MSE	0.016	0.001	0.001	0.014	0.002	0.001
TT	$H$	<b><u>0.211</u></b>	<b><u>0.320</u></b>	<b><u>0.316</u></b>	<b><u>0.217</u></b>	<b><u>0.288</u></b>	<b><u>0.288</u></b>
	MSE	0.008	0.002	0.001	0.007	0.001	0.000

Table 17: Estimations of  $H$  and MSEs for  $H = 0.4$  from three directions; under four different wavelet filters, in the non-contaminated case.

		Haar			Coiflet4		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b><u>0.334</u></b>	<b><u>0.375</u></b>	<b><u>0.375</u></b>	<b><u>0.372</u></b>	<b><u>0.413</u></b>	<b><u>0.414</u></b>
	$MSE$	0.005	0.002	0.002	0.002	0.002	0.002
AV	$H$	<b>0.254</b>	<b>0.353</b>	<b>0.354</b>	<b>0.311</b>	<b>0.342</b>	<b>0.340</b>
	$MSE$	0.021	0.003	0.003	0.008	0.004	0.004
TT	$H$	<b>0.278</b>	<b>0.361</b>	<b>0.362</b>	<b>0.329</b>	<b>0.352</b>	<b>0.350</b>
	$MSE$	0.015	0.002	0.002	0.005	0.003	0.003

  

		Daub6			Symmlet8		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b><u>0.369</u></b>	<b><u>0.401</u></b>	<b><u>0.406</u></b>	<b><u>0.373</u></b>	<b><u>0.421</u></b>	<b><u>0.411</u></b>
	$MSE$	0.002	0.002	0.001	0.002	0.002	0.002
AV	$H$	<b>0.306</b>	<b>0.385</b>	<b>0.387</b>	<b>0.313</b>	<b>0.350</b>	<b>0.349</b>
	$MSE$	0.009	0.001	0.001	0.008	0.003	0.003
TT	$H$	<b>0.326</b>	<b>0.393</b>	<b><u>0.395</u></b>	<b>0.331</b>	<b>0.360</b>	<b>0.358</b>
	$MSE$	0.006	0.001	0.001	0.005	0.002	0.002

Table 18: Estimations of  $H$  and MSEs for  $H = 0.4$  from three directions; under four different wavelet filters, in the contaminated case.

		Haar			Coiflet4		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b><u>0.437</u></b>	<b>0.478</b>	<b>0.474</b>	<b>0.476</b>	<b>0.516</b>	<b>0.520</b>
	$MSE$	0.003	0.008	0.007	0.007	0.015	0.016
AV	$H$	<b>0.267</b>	<b>0.368</b>	<b>0.368</b>	<b>0.324</b>	<b>0.355</b>	<b>0.351</b>
	$MSE$	0.018	0.001	0.001	0.006	0.002	0.003
TT	$H$	<b>0.303</b>	<b><u>0.386</u></b>	<b><u>0.387</u></b>	<b><u>0.353</u></b>	<b><u>0.375</u></b>	<b><u>0.372</u></b>
	$MSE$	0.010	0.001	0.001	0.002	0.001	0.001

  

		Daub6			Symmlet8		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b>0.465</b>	<b>0.511</b>	<b>0.509</b>	<b>0.472</b>	<b>0.511</b>	<b>0.520</b>
	$MSE$	0.006	0.015	0.014	0.007	0.015	0.016
AV	$H$	<b>0.318</b>	<b><u>0.403</u></b>	<b><u>0.406</u></b>	<b>0.326</b>	<b>0.361</b>	<b>0.363</b>
	$MSE$	0.007	0.001	0.001	0.006	0.002	0.002
TT	$H$	<b><u>0.348</u></b>	<b>0.422</b>	<b>0.425</b>	<b><u>0.353</u></b>	<b><u>0.380</u></b>	<b><u>0.382</u></b>
	$MSE$	0.003	0.001	0.001	0.002	0.001	0.001

Table 19: Estimations of  $H$  and MSEs for  $H = 0.6$  from three directions; under four different wavelet filters, in the non-contaminated case.

		Haar			Coiflet4		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b>0.556</b>	<b>0.584</b>	<b>0.575</b>	<b>0.604</b>	<b>0.510</b>	<b>0.512</b>
	$MSE$	0.003	0.002	0.003	0.001	0.009	0.008
AV	$H$	<b>0.506</b>	<b>0.574</b>	<b>0.572</b>	<b>0.559</b>	<b>0.383</b>	<b>0.381</b>
	$MSE$	0.009	0.001	0.002	0.002	0.051	0.052
TT	$H$	<b>0.522</b>	<b>0.579</b>	<b>0.575</b>	<b>0.571</b>	<b>0.396</b>	<b>0.393</b>
	$MSE$	0.006	0.001	0.002	0.001	0.046	0.047

  

		Daub6			Symmlet8		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b>0.595</b>	<b>0.507</b>	<b>0.506</b>	<b>0.597</b>	<b>0.518</b>	<b>0.519</b>
	$MSE$	0.001	0.010	0.010	0.001	0.008	0.008
AV	$H$	<b>0.558</b>	<b>0.484</b>	<b>0.481</b>	<b>0.557</b>	<b>0.400</b>	<b>0.400</b>
	$MSE$	0.002	0.014	0.015	0.002	0.044	0.044
TT	$H$	<b>0.570</b>	<b>0.492</b>	<b>0.490</b>	<b>0.568</b>	<b>0.410</b>	<b>0.410</b>
	$MSE$	0.001	0.012	0.013	0.001	0.039	0.040

Table 20: Estimations of  $H$  and MSEs for  $H = 0.6$  from three directions; under four different wavelet filters, in the contaminated case.

		Haar			Coiflet4		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b>0.653</b>	<b>0.666</b>	<b>0.668</b>	<b>0.702</b>	<b>0.617</b>	<b>0.607</b>
	$MSE$	0.004	0.007	0.007	0.012	0.002	0.002
AV	$H$	<b>0.516</b>	<b>0.585</b>	<b>0.586</b>	<b>0.572</b>	<b>0.402</b>	<b>0.382</b>
	$MSE$	0.007	0.001	0.001	0.001	0.045	0.051
TT	$H$	<b>0.543</b>	<b>0.599</b>	<b>0.598</b>	<b>0.594</b>	<b>0.424</b>	<b>0.405</b>
	$MSE$	0.003	0.001	0.001	0.000	0.036	0.041

  

		Daub6			Symmlet8		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b>0.696</b>	<b>0.606</b>	<b>0.598</b>	<b>0.700</b>	<b>0.621</b>	<b>0.616</b>
	$MSE$	0.011	0.002	0.002	0.011	0.002	0.001
AV	$H$	<b>0.570</b>	<b>0.495</b>	<b>0.493</b>	<b>0.571</b>	<b>0.423</b>	<b>0.408</b>
	$MSE$	0.001	0.012	0.012	0.001	0.034	0.040
TT	$H$	<b>0.593</b>	<b>0.513</b>	<b>0.511</b>	<b>0.593</b>	<b>0.443</b>	<b>0.430</b>
	$MSE$	0.000	0.008	0.009	0.000	0.027	0.032

Table 21: Estimations of  $H$  and MSEs for  $H = 0.7$  from three directions; under four different wavelet filters, in the non-contaminated case.

		Haar			Coiflet4		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b>0.664</b>	<b>0.675</b>	<b>0.682</b>	<b>0.709</b>	<b>0.517</b>	<b>0.527</b>
	$MSE$	0.002	0.003	0.003	0.001	0.035	0.032
AV	$H$	<b>0.620</b>	<b>0.676</b>	<b>0.680</b>	<b>0.670</b>	<b>0.346</b>	<b>0.362</b>
	$MSE$	0.007	0.002	0.002	0.001	0.130	0.121
TT	$H$	<b>0.634</b>	<b>0.678</b>	<b>0.683</b>	<b>0.680</b>	<b>0.361</b>	<b>0.377</b>
	$MSE$	0.005	0.002	0.002	0.001	0.120	0.110

  

		Daub6			Symmlet8		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b>0.711</b>	<b>0.517</b>	<b>0.516</b>	<b>0.703</b>	<b>0.533</b>	<b>0.527</b>
	$MSE$	0.002	0.036	0.036	0.001	0.030	0.031
AV	$H$	<b>0.676</b>	<b>0.486</b>	<b>0.485</b>	<b>0.670</b>	<b>0.382</b>	<b>0.377</b>
	$MSE$	0.001	0.047	0.048	0.001	0.106	0.109
TT	$H$	<b>0.687</b>	<b>0.496</b>	<b>0.495</b>	<b>0.679</b>	<b>0.395</b>	<b>0.389</b>
	$MSE$	0.001	0.043	0.044	0.001	0.098	0.101

Table 22: Estimations of  $H$  and MSEs for  $H = 0.7$  from three directions; under four different wavelet filters, in the contaminated case.

		Haar			Coiflet4		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b>0.762</b>	<b>0.768</b>	<b>0.771</b>	<b>0.810</b>	<b>0.626</b>	<b>0.628</b>
	$MSE$	0.005	0.007	0.007	0.014	0.008	0.008
AV	$H$	<b>0.632</b>	<b>0.691</b>	<b>0.693</b>	<b>0.684</b>	<b>0.373</b>	<b>0.373</b>
	$MSE$	0.005	0.001	0.001	0.000	0.112	0.113
TT	$H$	<b>0.658</b>	<b>0.702</b>	<b>0.705</b>	<b>0.703</b>	<b>0.399</b>	<b>0.399</b>
	$MSE$	0.002	0.001	0.001	0.000	0.096	0.097

  

		Daub6			Symmlet8		
		diagonal	horizontal	vertical	diagonal	horizontal	vertical
OLS	$H$	<b>0.808</b>	<b>0.611</b>	<b>0.616</b>	<b>0.802</b>	<b>0.633</b>	<b>0.628</b>
	$MSE$	0.013	0.010	0.010	0.012	0.007	0.007
AV	$H$	<b>0.688</b>	<b>0.499</b>	<b>0.496</b>	<b>0.685</b>	<b>0.402</b>	<b>0.395</b>
	$MSE$	0.000	0.042	0.043	0.000	0.094	0.098
TT	$H$	<b>0.706</b>	<b>0.519</b>	<b>0.515</b>	<b>0.704</b>	<b>0.425</b>	<b>0.418</b>
	$MSE$	0.000	0.034	0.036	0.000	0.080	0.084